Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Winter Examination-2022

Subject Name: Mathematical Methods-II

Subject Code: 5SC04MAM1 **Branch: M.Sc. (Mathematics)**

Semester: 4 **Date: 22/09/2022** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **[07]**

a. Write second form of Euler's equation. (01)

b. Prove that the shortest distance between two points in a plane is a straight line. (02)

c. Define Symmetric Kernel . (02)

d. Explain Isoperimetric problem. (02)

Q-2 **Attempt all questions** **[14]**

a. Show that geodesics on sphere of radius a are great circles. (08)

b. Solve the functional $I[y(x)] = \int_0^{\pi/2} (y''^2 - y^2 - x^2)\,dx$ with the conditions $y(0) = 0, y'(0) = 1, y\left(\frac{\pi}{2}\right) = 1, y'\left(\frac{\pi}{2}\right) = 0$. (06)

OR

Q-2 **Attempt all questions** **[14]**

a. If the functional $I[y(x)] = \int_{x_1}^{x_2} f(x, y, y')\,dx$ has the extremum value, then show that the integrand f satisfy the Euler's Equation (07)

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0\,.$$

b. Prove that the extremal of $I[y(x)] = \int_0^2 \frac{y'^2}{x}\,dx$ with $y(0) = 0$ and $y(2) = 1$ is parabola. (04)

c. Find the extremal of the function $I[y(x)] = \int_{x_1}^{x_2} (1 + x^2 y')y'\,dx$. (03)

Q-3 **Attempt all questions** **[14]**

a. Prove : (06)

$$\int_a^x \int_a^{x_n} \cdots \int_a^{x_2} f(x_1)\,dx_1 \cdots dx_n = \frac{1}{(n-1)!}\int_a^x (x-t)^{n-1} f(t)\,dt$$

b. Solve the integral equation: $y(x) = 3x^2 + \int_0^x \sin(x-t)\,y(t)\,dt$. (05)

	c.	Solve the functional $I[x(t),y(t)]=\int_0^1(2x+x'^2+y'^2)\,dt$ with initial conditions $x(0)=0, y(0)=1, x(1)=1.5$ and $y(1)=2$.	(03)

OR

Q-3			**[14]**
	a.	Solve the Integro differential equation : $y'+4y+5\int_0^x y(t)\,dt=e^{-x}$ where $y(0)=0$.	(06)
	b.	Show that $y(x)=xe^x$ is a solution of the integral equation $y(x)=sinx+2\int_0^x \cos(x-t)\,y(t)\,dt$.	(05)
	c.	Define Fredhom Integral equation and state Leibnitz's rule.	(03)

SECTION – II

Q-4		**Attempt the Following questions**	**[07]**
	a.	State Arzela's Theorem.	(01)
	b.	State the Hermite differential equation and reduce it to Sturn Liouville differential equation.	(02)
	c.	Let $y_1(x)$ and $y_2(x)$ be solutions of the homogeneous Fredholm integral equation $y(x)=\lambda\int_a^b K(x,t)\,y(t)\,dt$.Then show that $\alpha_1 y_1(x)+\alpha_2 y_2(x)$ is also a solution of given equation.	(02)
	d.	State the Bessel's differential equation and reduce it to Sturn Liouville differential equation.	(02)
Q-5		**Attempt all questions**	**[14]**
	a.	Let $X=C([a,b])$ or $L^p([a,b])$ where $1\le p\le\infty$. If $\{x_n\}$ is bounded sequence in X then show that it is bounded in $L^1([a,b])$.	(08)
	b.	Find the eigenvalues and corresponding eigen function of the integral equation $y(x)=\lambda\int_0^1(2xt-4x^2)y(t)\,dt$.	(06)

OR

Q-5		**Attempt all questions**	**[14]**
	a.	State and prove Fredholm's Alternative Theorem.	(07)
	b.	Solve $y(x)=x+\lambda\int_0^1 xt\,y(t)\,dt$ by Resolvent kernel method.	(07)
Q-6		**Attempt all questions**	**[14]**
	a.	Reduce the differential equation $x^2y''+xy'+(k^2x^2-n^2)y=0$ into Bessel's differential equation with substitution $t=kx$.	(07)
	b.	Reduce the differential equation $xy''+(1+2\lambda)y'+xy=0$ into Bessel's differential equation with substitution $x^\lambda y=z$.where $\lambda\in R$. and reduce it to Sturn Liouville differential equation.	(07)

OR

Q-6		**Attempt all Questions**	**[14]**
	a.	Find eigen values and corresponding eigenfunction of the equation $y''+\lambda y=0$on the interval $[0,c]$with boundary conditions $$y(0)=0=y'(c).$$	(07)
	b.	Convert $x^3y''+(5x^2-x^3)y'+4xy=0$ into Leguerre differential equation with substitution$x^2y=z$.where $\lambda\in R$ and reduce it to Sturn Liouville differential equation.	(07)